Extending Arnold invariants to higher dimensions via Vassiliev theory

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It is a classical fact that any sphere eversion must involve an odd number of quadruple point jumps. Motivated by this, we propose a new topological invariant for generic immersions of a closed oriented surface Σ into $\mathbb{R}^3$, intended to capture the occurrence of such a jump during the eversion. This invariant was introduced in our recent work [1]. It extends Arnold strangeness invariant for plane curves to a higher-dimensional setting, both in terms of domain and target. It is constructed as a sum of mapping degrees associated with triple points, and partially detects quadruple point jumps, being sensitive to four out of the five possible types of quadruple point jumps. Our method relies on ideas from Vassiliev theory, namely the study of discriminants and singularity stratifications. In addition to these properties, our work is also situated in a broader historical context: Arnold invariants for plane curves were revisited as invariants of real algebraic curves from a perspective related to Hilbert's 16th problem (Rokhlin, Viro), and Viro introduced the encomplexing version J^-. If time permits, we will also discuss how our work continues this line, aiming at a more topological and geometric approach to such extensions. This is a joint work with Noboru Ito (Shinshu University).

References

[1] Noboru Ito and Hiroki Mizuno, *Arnold strangeness of surface immersions*, arXiv:2506.10457 (2025).