Algebraic models of Poincaré embeddings and the unknotting condition

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Let P be a compact polyhedron of dimension m, W a closed oriented PL-manifold of dimension n, and $f : P \to W$ a PL-embedding. Take a regular neighborhood T of $f(P)$ in W and set $C := \overline{W - T}$ (so $C \simeq W - f(P)$). The following homotopy pushout square is a typical example of what is called a Poincaré embedding:

$$\begin{array}{ccc} \partial T & \longrightarrow & T \\ \downarrow & & \downarrow \\ C & \longrightarrow & W \end{array} \tag{1}$$

where all the maps are inclusions. Lambrechts-Stanley [1] proved that an algebraic model of Poincaré embedding in the sense of rational homotopy theory can be reconstructed from any algebraic model of the map $f : P \to W$ if $n \geq 2m + 3$ and $H^1(f)$ is injective. This model was applied to symplectic blow-ups. On the other hand, Wall and Hudson showed that for an r-connected embedding f, the homotopy type of the square (1) depends only on the homotopy type of f if $n \geq m + 3$ and $r \geq 2m - n + 2$. This latter inequality is called the unknotting condition. It is natural to expect that an algebraic model of the square also depends only on a model of the map f under the unknotting condition. Lambrechts-Stanley proved partial results such as reconstruction of a model of the inclusion $C \subset W$ from a model of f under the condition, but a model of the full square has not been constructed yet. In this talk, we will explain the speaker's recent approach to the reconstruction of an algebraic model of the square under the unknotting condition.

References

[1] P. Lambrechts and D. Stanley, *Algebraic models of Poincaré embeddings*, Algebr. Geom. Topol. **5** (2005), 135–182.